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COMMENT

A parity conserving model with spontaneous annihilation

N Inui, A Yu Tretyakov and H Takayasu
School of Information Sciences, Tohoku University, Aramaki, Aoba-ku, Sendai, 980-77, Japan

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Abstract. We propose a new model with the parity conservation of the total number of particles and show that this model belongs to the directed percolation universality class.

In a recent paper by Jensen [1], the relation between the universality class of a non-equibrium phase transition and the parity conservation of particle numbers was studied. Noting that among the few one-dimensional interacting particle systems, not belonging to the directed percolation (DP) universality class, such as branching annihilating random walk (BARW) with even offsprings [2, 3] and Grassberger's model [4, 5] are known systems with parity conservation of the total number of particles, Jensen shows how breaking the parity conservation by introducing spontaneous annihilation brings the otherwise non-DP process, the four-offspring BARW, in the DP universality class, and suggests that there is a link between non-DP universality and parity conservation. Here we bring an example of a parity-conserving process, belonging to DP universality class, implying that parity conservation in a system as a whole does not exclude DP universality.

Our simple model is defined as the following. Each particle creates one particle on the right-hand side with probability p or disappears with probability $1-p$ at each step (see figure 1). When a particle is created on an occupied site, it annihilates leaving an empty site. When the parity of the initial number of particles is even, parity is conserved, as one can easily check by considering different possibilities. Time-dependent simulation is a very effective method for locating the critical point and estimating dynamical critical exponents [3, 6]. We start from a pair of occupied nearest-neighbour sites and calculate the average number of particles $n(t)$ and the survival probability $P(t)$. For each value of p considered, we performed 1 000 000 trials making 1000 steps in each trial. Some of the results are shown in figures 2 and 3.

From the scaling theory for DP and other interacting particle systems, the behaviours of $n(t)$ and $P(t)$ are known to be governed by power laws at the critical point p_c as $t \to \infty$

$$P(t) \propto t^{-\delta} \qquad n(t) \propto t^{\eta} . \tag{1}$$

The critical point can be found as the p value for which $P(t)$ and $n(t)$ give a straight line in a log–log plot when plotted against time, and δ and η are given by the corresponding slopes. Our simulation gives $p_c = 0.80940 \pm 0.00005$ for the critical point, while the exponents values are found as $\delta = 0.159 \pm 0.05$, $\eta = 0.315 \pm 0.05$, very close to the DP values $\delta = 0.1597 \pm 0.0003$, $\eta = 0.314 \pm 0.0003$ [3].

The notable difference between our model and BARW with even offsprings is the process of annihilation. Our model includes a single annihilation process, while in BARW, particles

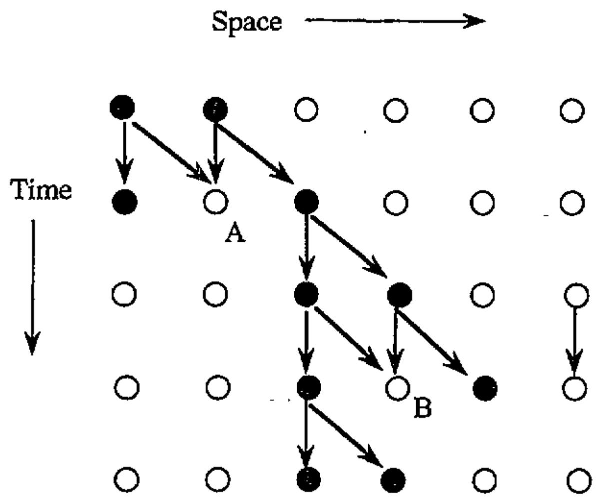

Figure 1. An illustration of the model. Full circles represents a particle. The annihilation by the occupation of two particles happens on points marked A and B.

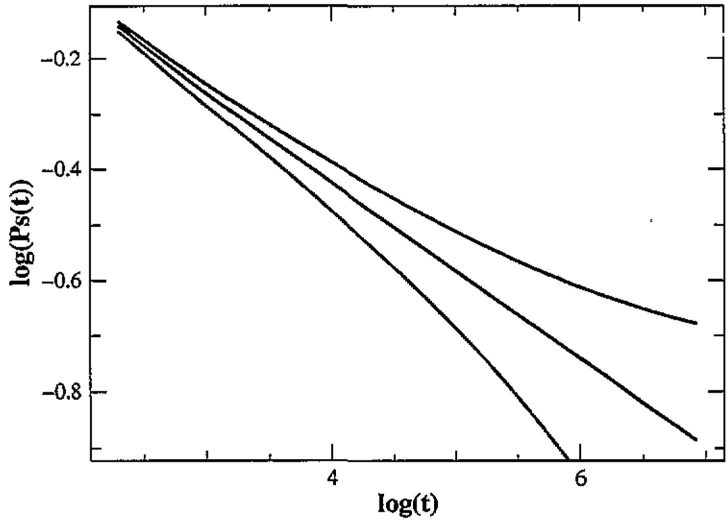

Figure 2. Logarithm of number of particles $n(t)$ versus $\log(t)$. From bottom to top, $p = 0.804$, 0.8094 and 0.814, respectively.

annihilate only in pairs. In even-offspring BARW parity is conserved locally, that is parity of a group of particles separated from the rest of the system by empty sites is conserved for as long as it remains separated, so that a group of particles with odd parity cannot become extinct by itself. In our model parity of a separate group of an odd number of particles in not conserved. Thus, while parity conservation in a system as a whole does not necessarily lead to non-DP universality, universality may by linked to local parity conservation.

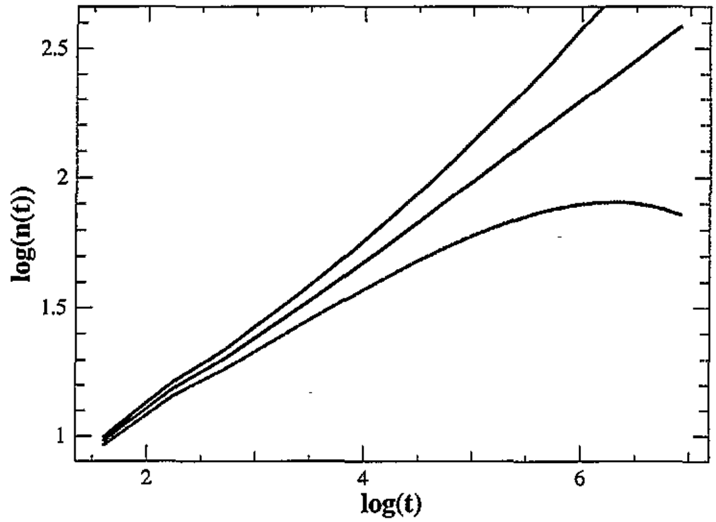

Figure 3. Logarithm of survival portability $P(t)$ versus $\log(t)$. From bottom to top, $p = 0.804$, 0.8094 and 0.814, respectively.

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References

[1] Jensen I 1993 *J. Phys. A: Math. Gen.* **26** 3921
[2] Takayasu H and Trtyakov A Yu 1992 *Phys. Rev. Lett.* **68** 3060
[3] Jensen I 1993 *Phys. Rev.* E **47** R1
[4] Grassberger P, Krause F and von der Twer T 1984 *J. Phys. A: Math. Gen.* **17** L105
[5] Grassberger P 1989 *J. Phys. A: Math. Gen.* **22** L1103
[6] Grassberger P and de la Torre A 1979 *Ann. Phys., NY* **122** 373